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GRAVITATIONAL FIELDS OF EARTH MODELS AND THE STRUCTURE OF THE EARTH'S INTERIOR

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PREFACE

Irregularities in the earth's gravitational field affect the motions of bodies orbiting the earth. The purpose of this study is to determine the effect upon the earth's gravitational field of assumed variations in mass distribution between oceanic and continental areas, using theoretical earth models. The results of this study may contribute to current endeavors to increase our knowledge of the nature of the earth's interior. Such knowledge is important because of its effect upon the earth's gravitational field, variations in which affect near-earth satellite orbits. A better understanding of the earth and its gravity field is also of interest to geodesists who are concerned with the shape of the earth and with improving the accuracy of world mapping.

SUMMARY

The third zonal harmonic (J_3) of the earth's potential field, the value of which has been determined from the orbits of near-earth satellites, contributes a pear-shaped component to its field configuration. The cause has not been definitely established.

The potential field can be expressed by surface-gravity anomalies. Gravity anomalies result from inhomogeneities in mass distribution within a body. In an earlier study(1) the gravitational fields of earth models were calculated to determine whether the distinct variations in mass distribution that exist between oceanic and continental crustal blocks could account for the third zonal harmonic. The results obtained from those models led to the construction of a second set of models; the results obtained from the second set are presented in this Memorandum.

The second set of earth models incorporates conventional crustal structures and an uppermost mantle structure based on seismic evidence regarding Gutenberg's low-velocity layer. The depths of isostatic compensation considered were 200 and 300 km. The models produced negative surface-gravity anomalies over continental blocks, whose magnitude increased slightly as depth to compensation increased. Corresponding to the general distribution of oceans and continents, these anomalies are comparable in magnitude and agree in sign with the satellite-derived anomaly.

The results of the study indicate that differences in density layering between oceanic and continental blocks may account for the third zonal harmonic, provided that the crust - upper-mantle structure has a double density-contrast crossover which results in a lower center of gravity under the continents; the depth of isostatic compensation occurs in the upper mantle at depths of at least 200 to 300 km; and the difference in seismic velocities in the low-velocity layer under oceans and continents may be due, in part, to small density differences between the two areas.

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CONTENTS

I. INTRODUCTION

In an earlier study[1] the gravitational fields of earth models were calculated to determine what effect the differences in density layering between oceanic and continental areas would have upon the field configuration. The interval considered was that between the earth's surface and depths of 33 to 70 km. Isostatic equilibrium was assumed, and standard oceanic and continental crustal sections were used. Over the continental area, the models produced a positive surface-gravity anomaly. Corresponding to the distribution of continents, such an anomaly is of opposite sign to the surface anomaly related to the third zonal harmonic (J_3), which is derived from the orbits of near-earth satellites. For details of background material covering the discussion of the earth's potential field expressed in spherical harmonics and its relation to surface-gravity anomalies, and of the geological conditions pertaining to the earth models, the reader is referred to Ref. 1.

It was concluded that the discrepancy seems to be most plausibly explained by mass inhomogeneities deeper within the upper mantle. This Memorandum discusses the results obtained from the gravitational fields of a second set of earth models where the interval between the earth's surface and depths of 200 to 300 km is considered.

II. MODEL STRUCTURE

In this study we are interested only in the gravity contrast contributed by differences in oceanic and continental mass distributions. Therefore, the effects on the earth's gravitational field due to the earth's rotation and oblateness, the ellipticity of the equator, and the attraction of other nearby bodies such as the sun and moon were excluded. Complete isostatic equilibrium was assumed, which means that at a "depth of compensation" D, hydrostatic equilibrium prevails; thus at D any rock unit is under the same pressure, regardless of whether the unit is under mountain, lowland, or ocean.[2]

Spherical earth models 6371 km in radius and incorporating typical oceanic and continental crustal sections were constructed. The depths of compensation used were 200 and 300 km. Although assumptions of physical conditions in the earth's interior become more speculative with depth, it is believed that the relative values used in these models are a plausible interpretation of upper-mantle structure based on certain seismic evidence. Between the approximate depths of 100 to 250 km[3] there is a layer through which seismic waves travel more slowly than normal depth-extrapolation methods predict. This layer is called Gutenberg's low-velocity layer, after the man who first described it.[4] Its existence has been known for some time, but its worldwide extent and nonuniform nature have only recently been determined by the accumulation of sufficient seismic data. The low-velocity layer is different under oceans and continents. The difference has been interpreted by some investigators[5] as meaning that the layer is shallower under the ocean, and by others,[6] that the layer occurs at the same depth under ocean and continent but that the velocity under the ocean is lower. The models were based on the latter interpretation.

The standard oceanic and continental crustal sections have a density-contrast crossover point at a depth equivalent to that of the base of the oceanic sediments. Above this depth, the density is greater for the continental section; below it, the density for the continent is always less, down to a depth of compensation at the base of the crust or a short distance below it. In the spherical body, this

type of structure ensures a positive anomaly over the continental section. However, if upper mantle material beneath the continent were slightly more dense than that beneath the ocean, and if isostatic compensation were maintained at a lower depth, a negative gravity anomaly over the continent could result. This means that the crust - upper-mantle density layering structure would have two density-contrast crossover points: one at the base of the oceanic sediments and one near the top of the low-velocity layer. This type of structure, which is thought to be compatible with the velocity-differential interpretation of the low-velocity layer, is tested in the earth models.

The density values and distribution of the oceanic and continental sections, labeled O and C respectively, used in the models are shown in Fig. 1. Core and mantle densities appear in Table 1.

The gravitational attractions of each of the resultant models were calculated by means of the formula(7)

$$g = 4/3 \, \pi \, G \sum \rho \frac{(b^3 - a^3)}{r^2}$$

where g is the gravitational attraction; G is the gravitational constant; ρ is the average density; b and a are the distances from the center to the outer and inner surfaces, respectively, of spherical shells; and r is the distance from the center of the sphere to the point at which the attraction of gravity is calculated ($r \geq b$).

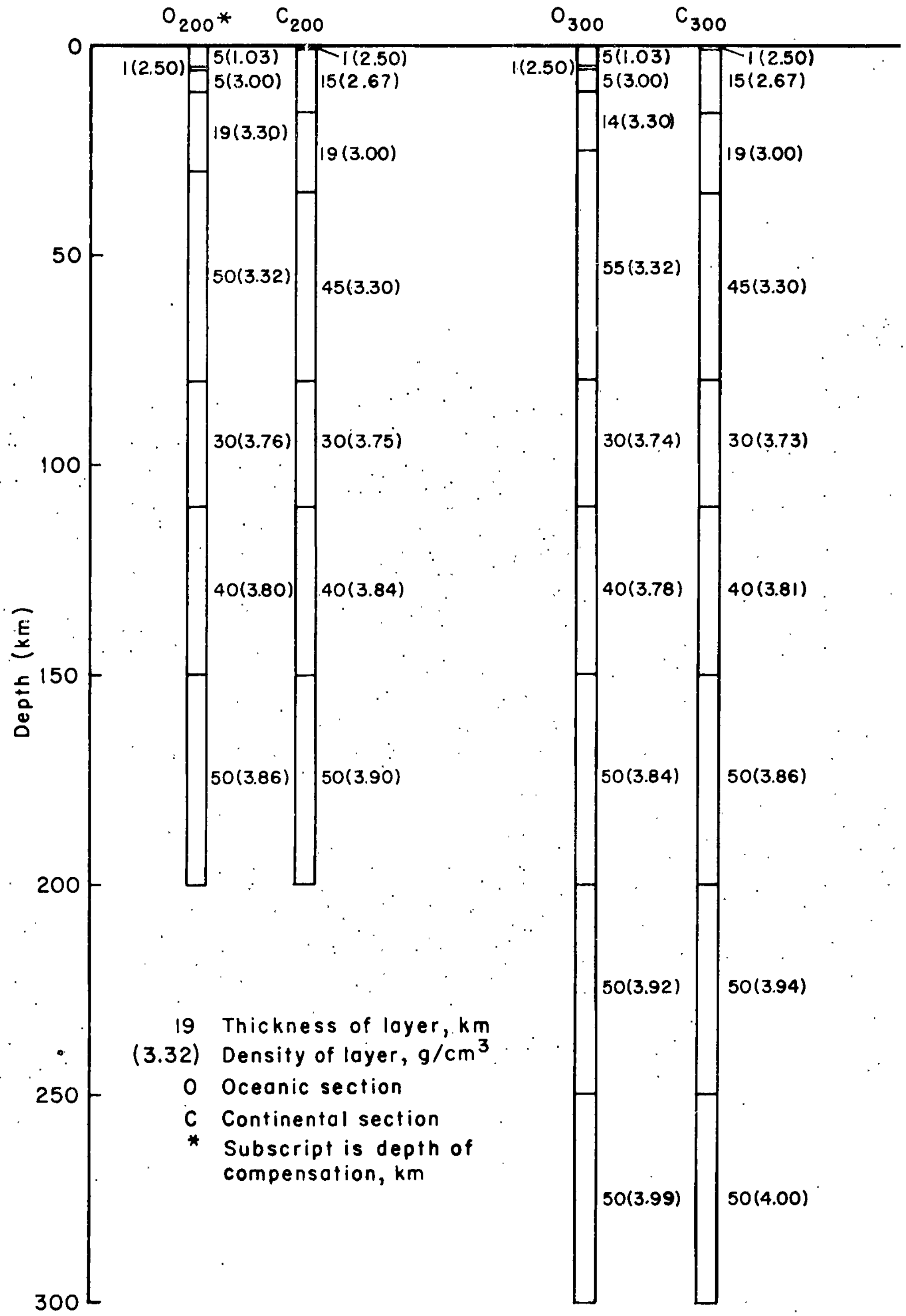

Fig. 1— Density values for models of oceanic and continental sections

Table 1

DENSITY (AFTER BULLEN), GRAVITY, AND PRESSURE IN THE EARTH[5]

Depth (km)	Radius, r (km)	Density, ρ (g/cm^3)	Gravity, g (cm/sec^2)	Pressure, p (bars x 10^6)[a]
33	6338	3.32	983	0.009
80	6291	3.36	984	0.025
80	6291	3.87	984	0.025
200	6171	3.94	983	0.071
400	5971	4.06	981	0.149
600	5771	4.18	979	0.230
800	5571	4.30	977	0.313
1000	5371	4.41	975	0.398
1200	5171	4.52	974	0.485
1400	4971	6.63	975	0.574
1600	4771	4.74	977	0.666
1800	4571	4.84	980	0.759
2000	4371	4.94	987	0.855
2200	4171	5.03	996	0.954
2400	3971	5.13	1010	1.056
2600	3771	5.22	1028	1.161
2700	3671	5.27	1041	1.216
2900	3471	5.57	1068	1.330
2900	3471	9.74	1068	1.33
3000	3371	9.90	1048	1.41
3200	3171	10.20	1005	1.62
3400	2971	10.47	960	1.82
3600	2771	10.72	913	2.02
3800	2571	10.95	865	2.21
4000	2371	11.16	816	2.40
4200	2171	11.36	767	2.58
4400	1971	11.54	717	2.75
4600	1771	11.71	670	2.91
4800	1571	11.85	632	3.06
4982	1389	12.00	598	3.19
5121	1250	15.01	564	3.30
5400	971	16.16	457	3.53
5700	671	17.07	326	3.72
6000	371	17.65	184	3.85
6371	0	17.9	0	3.92

[a]The bar is the meteorologist's bar of 10^6 $dynes/cm^2$, which is equal to 0.987 atm. The pressure at the center is thus 3.92×10^{12} cgs units.

III. MODEL RESULTS AND DISCUSSION

Letting the value of the gravitational attraction for the oceanic section be used as a reference in defining the surface-gravity anomalies ($g_o = 0$), the resulting anomalies, Δg, for the continental sections are $C_{200} = -7.6$ mgl (milligals) and $C_{300} = -11.0$ mgl. (The subscripts of C indicate the depth to compensation in kilometers.) In both Model C_{200} and Model C_{300}, Δg is negative and increases slightly as depth to compensation increases.

In comparing the model anomalies with the satellite-derived anomalies, a slight adjustment in magnitude is necessary because the latter value includes an earth-rotation factor. The derivation of the gravitational field from the potential[8] gives

$$g_n = -(n - 1)(GMR^{-2})J_n$$

where the rotation factor is included, and for the model cases

$$\Delta g = -(n + 1)(GMR^{-2})J_n$$

where the rotation factor is excluded. Thus, for $n = 3$, Δg will be equivalent to twice g_3:

$$g_3 = -2(GMR^{-2})J_3 = -2(985)(-2.4 \times 10^{-6}) \text{ cm sec}^{-2} \approx 4.8 \text{ mgl}$$

$$\Delta g = -4(GMR^{-2})J_3 \approx 9.6 \text{ mgl}$$

Results from Models C_{200} and C_{300} give anomalies which are comparable in magnitude and agree in sign with the satellite-derived anomaly. The model values are expected to give only rough approximations of Δg because (1) isostatic equilibrium is not complete, (2) the earth is not a perfectly smooth sphere, and (3) the rock layers in the earth are not concentric shells of uniform density. The sign of the satellite-derived anomaly varies latitudinally depending upon

the sign of the P_n term. When evaluating for n = 3, P_3 is negative in the midlatitudes of the Northern Hemisphere. Therefore, the g_3 anomaly is negative in the midlatitudes of the Northern Hemisphere where the continental areas are concentrated. The model anomalies are also negative over the continental block. Thus, Models C_{200} and C_{300} demonstrate that a crust - upper-mantle structure having a double density-contrast crossover and a deep depth of compensation produces a gravitational field consistent with the third zonal harmonic component of the satellite-derived field.

However, it should be emphasized that a given gravity anomaly has no unique solution, and thus the actual density values used in the models are not to be taken as the only valid, or even the most realistic, solution in satisfying all the physical conditions. For example, a lower density contrast in the low-velocity layer coupled with an even greater depth of compensation would give the same results.

IV. CONCLUDING REMARKS

If the assumptions used in this Memorandum are valid, then certain conclusions and inferences can be made from the earth models studied:

- Differences in density layering between oceanic and continental blocks may account for the third zonal harmonic, provided that the crust - upper-mantle structure has a double density-contrast crossover which results in a lower center of gravity under the continents.
- Given the above condition, the depth of isostatic compensation occurs in the upper mantle at depths of at least 200 to 300 km, or even greater if the density contrasts in Gutenberg's low-velocity layer are less than those used in the models.
- The models support Aki and Press' interpretation of the low-velocity layer[6] and suggest that the difference in seismic velocities in that layer under oceans and continents may be due, in part, to small density differences between the two areas.

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